GRADE 10
MATHEMATICS
UNIT 4: RATIONAL AND RADICAL FUNCTIONS

RATIONAL FUNCTION

A **rational function** is a fraction in which the numerator and/or the denominator are polynomials. R(x) is a rational function if R(x) = p(x) / q(x) where p(x) and q(x) are both polynomials.

Example 1:

The function $f(x) = \frac{2x^4+3x^2-1}{x^2+1}$ is a rational function since the numerator, $2x^4 + 3x^2 - 1$, is a polynomial and the denominator, $x^2 + 1$, is also a polynomial.

NOTE*: Any constant is a polynomial.

Example 2:

The function $f(x) = \frac{2}{(x-2)(x^2+1)}$ is a rational function since the numerator, 2, is a constant and a polynomial and the denominator, $(x - 2)(x^2 + 1)$, is in factor form and also a polynomial.

NOT RATIONAL FUNCTION

- Square root ($\sqrt{x}$) is not a polynomial since the exponent of x is not an integer.
- Inverse or negative power ($\frac{1}{x}$ or x^{-1}) is not a polynomial since the exponent of x is not a non-negative integer.

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RADICAL FUNCTION

A **radical function** is a function that contains a square root ($\sqrt{x}$).

Example:

The function $f(x) = \sqrt{2x + 3}$ is a radical function.

EVALUATING IN RATIONAL FORM AND RADICAL FORM

Example:

Evaluate the expression $16^{3/2}$

SOLUTION

a. Rational Exponent Form:

$$16^{3/2} = (16^{1/2})^3 = 4^3 = 64$$

Radical Form:

$$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$$

TRY THIS!

Evaluate each expression in rational form and radical form.

1. $4^{\frac{5}{2}}$
2. $9^{-\frac{1}{2}}$
3. $1^{\frac{7}{8}}$

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PROPERTIES OF RATIONAL EXPONENTS

Property	Definition	Example
Product of Power	$a^m \bullet a^n = a^{m+n}$	$5^{1/2} \bullet 5^{3/2} = 5^{1/2+3/2} = 5^2 = 25$
Power of Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{\frac{5}{2}\times 2} = 3^5 = 243$
Power of Product	$(ab)^m = a^m b^m$	$(16 \bullet 9)^{1/2} = 16^{1/2} \bullet 9^{1/2} = 4 \bullet 3 = 12$
Quotient of Power	$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{5/2-1/2} = 4^2 = 16$
Power of Quotient	$(\frac{a}{b})^m = \frac{a^m}{b^m},\ b \neq 0$	$(\frac{27}{64})^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{(3^3)^{1/3}}{(4^3)^{1/3}} = \frac{3}{4}$
Negative Exponent	$a^{-m} = \frac{1}{a^m},\ a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1,\ a \neq 0$	$213^0 = 1$

TRY THIS!

Simplify the expression.

4. $2^{\frac{3}{4}} \bullet 2^{\frac{1}{2}}$
5. $\frac{3}{3^{\frac{1}{4}}}$
6. $(\frac{20^{\frac{1}{2}}}{5^{\frac{1}{2}}})^3$

SOLVING RATIONAL EXPRESSION

A rational expression can be solved using **cross multiplication**.

Example:

Solve $\frac{3}{x+1} = \frac{9}{4x+5}$.

$$\frac{3}{x+1} = \frac{9}{4x+5}$$

$\Rightarrow 3(4x + 5) = 9(x + 1)$
$\Rightarrow 12x + 15 = 9x + 9$
$\Rightarrow 12x - 9x = 9 - 15$
$\Rightarrow 3x = -6$
$\Rightarrow x = -\frac{6}{3}$
$\therefore x = -2$

TRY THIS!

Solve the expression.

7. $\frac{3}{5x} = \frac{2}{x-7}$
8. $\frac{1}{2x+5} = \frac{2x+5}{x-3}$

PROPERTIES OF RADICAL EXPONENTS

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \bullet b} = \sqrt[n]{a} \bullet \sqrt[n]{b}$	$\sqrt[3]{4} \bullet \sqrt[3]{2} = \sqrt[3]{4 \bullet 2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}},\ b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

TRY THIS!

Simplify the expression.

9. $\sqrt[4]{27} \bullet \sqrt[4]{3}$
10. $\frac{\sqrt[3]{250}}{\sqrt[3]{2}}$
11. $7\sqrt[5]{12} - \sqrt[5]{12}$

SOLVING RADICAL EXPRESSION

- **Step 1**: Isolate the radical on one side of the equation, if necessary.
- **Step 2**: Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.
- **Step 3**: Solve the resulting equation.

Example:

Solve $\sqrt[3]{2x - 9} - 1 = 2$.

$\sqrt[3]{2x - 9} - 1 = 2$

$\Rightarrow \sqrt[3]{2x - 9} = 2 + 1$

$\Rightarrow \sqrt[3]{2x - 9} = 3$

$\Rightarrow (\sqrt[3]{2x - 9})^3 = 3^3$ (Taking cube on both sides)

$\Rightarrow 2x - 9 = 27$

$\Rightarrow 2x = 27 + 9$

$\Rightarrow 2x = 36$

$\Rightarrow x = \frac{36}{2}$

$\therefore \ x = 18$

TRY THIS!

Solve the expression.

12. $\sqrt[3]{x} - 9 =- 6$
13. $\sqrt{x - 25} = 2$

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VARIABLE EXPRESSION

	Rule	Example
When n is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} =- 5$
When n is even	$\sqrt[n]{x^n} = \lvert x\rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Example:
Simplify the expressions.

a. $\sqrt[3]{64y^6}$ **b.** $\sqrt[4]{\frac{x^4}{y^8}}$

a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \bullet \sqrt[3]{(y^2)^3} = 4y^2$

b. $\sqrt[4]{\frac{x^4}{y^8}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{y^8}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{(y^2)^4}} = \frac{\lvert x\rvert}{y^2}$

TRY THIS!

Simplify the expression.

14. $\sqrt[3]{27q^9}$
15. $\sqrt[5]{\frac{x^{10}}{y^5}}$
16. $\sqrt{9w^5} - w\sqrt{w^3}$

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QUICK REVIEW: RATIONAL AND RADICAL FUNCTIONS

Sl. no.

1. RATIONAL FUNCTION

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SOLVING RATIONAL EXPRESSION

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2. RADICAL FUNCTION

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PROPERTIES OF RADICAL EXPONENTS

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SOLVING RADICAL EXPRESSION

- **Step 1**: Isolate the radical on one side of the equation, if necessary.
- **Step 2**: Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.
- **Step 3**: Solve the resulting equation.

3. **EVALUATING IN RATIONAL FORM AND RADICAL FORM**

Evaluate each expression.

a. $16^{3/2}$ **b.** $32^{-3/5}$

SOLUTION

Rational Exponent Form

a. $16^{3/2} = (16^{1/2})^3 = 4^3 = 64$

b. $32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3} = \frac{1}{2^3} = \frac{1}{8}$

Radical Form

$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$

$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3} = \frac{1}{2^3} = \frac{1}{8}$

4. **VARIABLE EXPRESSION**

	Rule	Example
When n is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When n is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$