GRADE 10
MATHEMATICS
UNIT 5: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EXPONENTIAL FUNCTION

Exponential function can be defined by the formula $y = ab^x$, where $a \neq 0$, and base b is any positive real number other than 1 ($b \neq 1$).

Natural exponential function is the most commonly used exponential function base denoted by the transcendental number "e", which is approximately equal to 2.71828.

PROPERTIES OF EXPONENTS		
Sl. no.	Rule	Example
1.	$a^m a^n = a^{m+n}$	$2^2 \bullet 2^3 = 2^{2+3} = 32$
2.	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$
3.	$(a^m)^n = a^{mn}$	$(2^3)^2 = 2^{3\times2} = 64$
4.	$(ab)^n = a^n b^n$	$(2 \bullet 3)^2 = 2^2 \times 3^2 = 36$
5.	$a^{-n} = \frac{1}{a^n}$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
6.	$(\frac{a}{b})^n = \frac{a^n}{b^n}$	$(\frac{1}{2})^2 = \frac{1^2}{2^2} = \frac{1}{4}$
7.	$(\frac{a}{b})^{-n} = \frac{b^n}{a^n}$	$(\frac{1}{2})^{-2} = \frac{2^2}{1^2} = 4$
8.	$a^0 = 1$ (unless $a = 0$) $e^0 = 1$	$Any\ number^0 = 1$
9.	$e^x \times e^y = e^{x+y}$	$e^3 \times e^2 = e^{3+2} = e^5 = 148.413$
10.	$\frac{e^x}{e^y} = e^{x-y}$	$\frac{e^3}{e^2} = e^{3-2} = e^1 = 2.718$
11.	$e^{-x} = \frac{1}{e^x}$	$e^{-2} = \frac{1}{e^2} = \frac{1}{(2.718)^2} = 0.135$
12.	$(e^x)^y = e^{xy}$	$(e^3)^2 = e^6 = 403.429$

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TRY THIS!

Evaluate the expression.

1. $y = 4^{x}$, when $x = 2$
2. $y = 3(\frac{2}{3})^{x}$, when $x = 3$
3. $y = \frac{2^{3x}}{2^{2x}}$, when $x = 1$
4. $y = e^{-x}$, when $x = 0$
5. $y = 2e^{x} - 1$, when $x = 4$
6. $y = e^{x} \times e^{y}$, when $x = 2$

Simplify the expression.

7. $4^{2} \bullet 4^{3}$
8. $\frac{3^{3}}{3^{3}}$
9. $(\frac{3}{2})^{-2}$
10. $(2 \bullet 4)^{2}$
11. $e^{5} \times e^{2}$
12. $\frac{e^{2}}{e^{2}}$

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EXPONENTIAL GROWTH FUNCTION

Exponential growth or exponential increase is defined by the formula $y = ab^x$, when $a > 0$ and $b > 1$.

EXPONENTIAL DECAY FUNCTION

Exponential decay or exponential decrease is defined by the formula $y = ab^x$, when $a > 0$ and $0 < b < 1$. Here, b is called the decay factor.

Example:

Tell whether the functions represent exponential growth or exponential decay.

a. $y = 2^x$ **b.** $y = (\frac{1}{2})^x$

a. Since, base $= 2 > 1$, so the function represents exponential growth.

b. Since, base $= 0 < \frac{1}{2} < 1$, so the function represents exponential decay.

TRY THIS!

Tell whether the functions represent exponential growth or exponential decay.

13. $y = 4^x$
14. $y = (\frac{2}{3})^x$
15. $y = (1.5)^x$

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LOGARITHMIC FUNCTION

Logarithmic function can defined by the formula $log_b y = x$ (log base b of y), if and only if $b^x = y$, where $b \neq 1$. Therefore, $log_b y = x$ and $b^x = y$ are equivalent, and $log_b y = x$ is in logarithmic form whilst $b^x = y$ is in exponential form.

Natural logarithmic function is the logarithmic function of base e denoted by $log_e x$ or ln_x. The difference between log and In is that log is defined for base 10 and In is denoted for base e.

PROPERTIES OF LOGARITHMS		
Sl. no.	Rule	Example
1.	Product Rule: *$log_b mn = log_b m + log_b n$ *$ln\, xy = ln\, x + ln\, y$	* $log_2 21 = log_2(3 \bullet 7) = log_2 3 + log_2 7$ *$ln\, 21 = ln\,(3 \bullet 7) = ln\, 3 + ln\, 7 = 3.045$
2.	Quotient Rule: *$log_b \frac{m}{n} = log_b m - log_b n$ *$ln\, \frac{x}{y} = ln\, x - ln\, y$	*$log_2 \frac{3}{7} = log_2 3 - log_2 7$ *$ln\, \frac{3}{7} = ln\, 3 - ln\, 7 =- 0.847$
3.	Power Rule: *$log_b m^n = nlog_b m$ *$ln\, x^y = y\, ln\, x$	*$log_2 49 = log_2 7^2 = 2log_2 7$ *$ln\, 49 = ln\, 7^2 = 2\, ln\, 7 = 3.892$, or $ln\, 49 = 3.892$ (using calculator)
4.	Base Change Rule: $log_c a = \frac{log_b a}{log_b c}$	$log_2 3 = \frac{log3}{log2}$
5.	Equality Rule: *If $log_b m = log_b n$, then $m = n$ *If $ln\, x = ln\, y$, then $x = y$	*$log_2 3x = log_2 6$, $\Rightarrow 3x = 6$ $\Rightarrow x = \frac{6}{3}$ $\therefore x = 2$ *$ln\, 3x = ln\, 6$ $\Rightarrow 3x = 6$ $\Rightarrow x = \frac{6}{3}$ $\therefore x = 2$

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	*If $e^{x} = e^{y}$, then $x = y$	*$e^{3x} = e^{6}$ $\Rightarrow 3x = 6$ $\Rightarrow x = \frac{6}{3}$ $\therefore x = 2$
6.	$log_{b}1 = 0$	$log_{10}1 = 0$
7.	$log_{b}b = 1$	$log_{10}10 = 1$
8.	$log_{a}b = \frac{1}{log_{b}a}$	$log_{2}10 = \frac{1}{log_{10}2}$
9.	$log_{b}b^{x} = xlog_{b}b = x \bullet 1 = x$	$log_{10}10^{2} = 2log_{10}10 = 2 \bullet 1 = 2$
10.	$e^{ln\, x} = x$	$e^{ln\, 2} = 2$
11.	$ln\, e^{x} = x\, ln\, e = x \bullet 1 = x$	$ln\, e^{3} = 3\, ln\, e = 3 \bullet 1 = 3$
12.	$ln\, e = 1$	

TRY THIS!

Evaluate the logarithms assuming $log_6 5 \approx 0.898$ and $log_6 8 \approx 1.161$.

16. $log_6 \frac{5}{8}$
17. $log_6 64$

Evaluate the logarithm using (a) common property, and (b) natural property.

18. $log_3 8$

Expand the logarithms.

19. $log_6 3x^4$
20. $\frac{ln\, e^3}{ln\, e}$

Condense the logarithms.

21. $log\, x - log\, 9$
22. $log\, 9 + 3\, log\, 9$
23. $ln\, 4 + 3\, ln\, 3 - ln 12$

Solve the logarithms.

24. $log\, 2x + log\, (x - 5)$
25. $ln(7x - 4) = ln\, (2x - 11)$

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QUICK REVIEW: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

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6.	$log_b 1 = 0$	$log_{10} 1 = 0$
7.	$log_b b = 1$	$log_{10} 10 = 1$
8.	$log_a b = \frac{1}{log_b a}$	$log_2 10 = \frac{1}{log_{10} 2}$
9.	$log_b b^x = xlog_b b = x \bullet 1 = x$	$log_{10} 10^2 = 2log_{10} 10 = 2 \bullet 1 = 2$
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