GRADE 10
MATHEMATICS
UNIT 6: SEQUENCE AND SERIES

SEQUENCE

A **sequence** can be defined as an ordered list of numbers sequentially.

- **Finite sequence** is a function with limited or countable number of terms with domain of first n positive integers {1, 2, 3, …, n} and range $\{a_1, a_2, a_3, ..., a_n\}$.

  Example: $2, 4, 6, 8$

- **Infinite sequence** is a function with limitless or uncountable number of terms with domain of all positive integers {1, 2, 3, …, ∞} and range $\{a_1, a_2, a_3, ..., a_\infty\}$.

  Example: $2, 4, 6, 8, ...$

NOTE*: The domain of a sequence may begin with 0 instead of 1. When this is the case, the domain of a finite sequence is the set {0, 1, 2, 3, . . . , n} and the domain of an infinite sequence becomes the set of nonnegative integers. Unless otherwise indicated, assume the domain of a sequence begins with 1.

Example: **Find the first three terms of the sequence $a_n = 2n + 5$ starting with a_0.**

$a_0 = 2(0) + 5 = 0 + 5 = 5$

$a_1 = 2(1) + 5 = 2 + 5 = 7$

$a_2 = 2(2) + 5 = 4 + 5 = 9$ Ans: 5, 7 and 9.

TRY THIS!

Find the first three terms of the sequence.

1. $a_n = 2n + 5$
2. $f(n) = (-2)^{n-1}$

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SERIES

A **series** can be defined as a resulting expression when the terms of a sequence are added together.

- **Finite series** is a function with sum of limited or countable number of terms expressed as $\sum_{i=1}^{n} a_i$.

  Example: $2 + 4 + 6 + 8 = \sum_{i=1}^{n} 2i$

- **Infinite series** is a function with sum of limitless or uncountable number of terms expressed as $\sum_{i=1}^{\infty} a_i$.

  Example: $2 + 4 + 6 + 8 + ... = \sum_{i=1}^{\infty} 2i$

SIGMA NOTATION FOR SERIES

For both finite and infinite series, the index of summation is i and the lower limit of summation is 1. The upper limit of summation is 4 for the finite series and ∞ (infinity) for the infinite series. Summation notation is also called sigma notation because it uses the uppercase Greek letter sigma, written $\sum$.

Example 1: **Write the series** $25 + 50 + 75 + ... + 250$ **using sigma notation** $\sum$**.**

Notice that the first term is $25(1)$, second term is $25(2)$, third term is $25(3)$, and the last term is $25(10)$, so the terms of the series can be written as $a_i = 25i,\ where\ i = 1, 2, 3, ..., 10$.

Ans: $25 + 50 + 75 + ... + 250 = \sum_{i=1}^{10} 25i$

Example 2: **Write the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} +$... using sigma notation $\sum$.**

Notice that for each term the denominator is one more than numerator, so the terms of the series can be written as $a_i = \frac{i}{i+1}$, where $i = 1, 2, 3, 4,$...

Ans: $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + ... = \sum_{i=1}^{\infty} \frac{i}{i+1}$

SUM OF SERIES

A series of numbers can be added beginning from the lower limit till the upper limit.

Example 1: **Find the sum of the series $\sum_{i=4}^{8} (3 + i^2)$.**

$$\sum_{i=4}^{8} (3 + i^2) = (3 + 4^2) + (3 + 5^2) + (3 + 6^2) + (3 + 7^2) + (3 + 8^2)$$

$$= 19 + 28 + 39 + 52 + 67$$

$$= 205$$

Ans: $\sum_{i=4}^{8} (3 + i^2) = 205$

Formulae for Special Series	
Sum of n terms of 1	$\sum_{i=1}^{n} 1 = n$
Sum of first n positive integers	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
Sum of squares of first n positive integers	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Example 2: **Find the sum of the series $\sum_{i=1}^{7} i^2$.**

$$\sum_{i=1}^{7} i^2 = 1^2 + 2^2 + 3^2 + \ldots + 7^2 = \frac{7(7+1)(2\times7+1)}{6} = \frac{7\times8\times15}{6} = \frac{840}{6} = 140$$

Ans: $\sum_{i=1}^{7} i^2 = 140$

TRY THIS!

Find the sum of the series.

3. $\sum_{i=1}^{5} 8i$

4. $\sum_{i=3}^{7} (i^2 - 1)$

5. $\sum_{i=1}^{34} 1$

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ARITHMETIC SEQUENCE

In **arithmetic sequence**, the common difference (denoted by 'd') or difference between consecutive terms is constant.

Example 1:

Tell whether each sequence is arithmetic:

a. $-9, -2, 5, 12, 19, ...$ **b.** $23, 15, 9, 5, 3, ...$

SOLUTION

a. Finding the common difference:

$a_2 - a_1 = -2 - (-9) = 7$

$a_3 - a_2 = 5 - (-2) = 7$

$a_4 - a_3 = 12 - 5 = 7$

$a_5 - a_4 = 19 - 12 = 7$

Since, the common difference is constant, it is an arithmetic sequence.

b. Finding the common difference:

$a_2 - a_1 = 15 - 23 = -8$

$a_3 - a_2 = 9 - 15 = -6$

$a_4 - a_3 = 5 - 9 = -4$

$a_5 - a_4 = 3 - 5 = -2$

Since, the common difference is not constant, it is not an arithmetic sequence.

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TRY THIS!

Identify the arithmetic sequence.

6. $2, 5, 8, 11, 14,...$

7. $8, 4, 2, 1, \frac{1}{2},...$

Nth TERM OF ARITHMETIC SEQUENCE

The n^{th} term of arithmetic sequence with first term a_1 and common difference d is given by: $a_n = a_1 + (n - 1)d$.

Example 2:

a. Write a rule for nth term of the arithmetic sequence $3, 8, 13, 18,...$

b. Find a_{15}**.**

a. A/Q,

$a_1 = 3, d = 5$

Now,

$a_n = a_1 + (n - 1)d$

$= 3 + (n - 1)5$

$= 3 + 5n - 5$

$= 5n - 2$

Ans: $a_n = 5n - 2$

b. $a_{15} = 5(15) - 2 = 30 - 2 = 28$

Ans: $a_{15} = 28$

TRY THIS!

Write a rule for the nth term of the arithmetic sequence, and find a_{25}.

8. $7, 11, 15, 19, ...$

SUM OF ARITHMETIC SEQUENCE

The expression formed by adding arithmetic sequences is called arithmetic series. Sum of the first n terms of an arithmetic series is denoted by S_n.

Sum of finite arithmetic series: $S_n = n(\frac{a_1 + a_n}{2})$

Example 3:

Find the sum of arithmetic series $\sum_{i=1}^{20} (3i + 7)$.

A/Q,

$a_1 = 3(1) + 7 = 3 + 7 = 10$

$a_{20} = 3(20) + 7 = 60 + 7 = 67$

Now,

$S_n = n(\frac{a_1 + a_n}{2})$

$S_{20} = 20(\frac{10+67}{2}) = 20 \times \frac{77}{2} = 770$

Ans: $S_{20} = 770$

TRY THIS!

Find the sum of the arithmetic series.

9. $\sum_{i=1}^{10} 9i$

10. $\sum_{i=1}^{12} (7i + 12)$

GEOMETRIC SEQUENCE

In **geometric sequence**, the ratio (denoted by 'r') or common ratio of any term to the previous term is constant.

Example 1:

Tell whether each sequence is geometric:

a. $256, 64, 16, 4, 1, \ldots$ **b.** $6, 12, 20, 30, 42, \ldots$

SOLUTION

a. Finding the common ratio:

$$\frac{a_2}{a_1} = \frac{64}{256} = \frac{1}{4}$$

$$\frac{a_3}{a_2} = \frac{16}{64} = \frac{1}{4}$$

$$\frac{a_4}{a_3} = \frac{4}{16} = \frac{1}{4}$$

$$\frac{a_4}{a_3} = \frac{1}{4}$$

Since, the common ratio is constant, it is a geometric sequence.

b. Finding the common ratio:

$$\frac{a_2}{a_1} = \frac{12}{6} = 2$$

$$\frac{a_3}{a_2} = \frac{20}{12} = \frac{5}{3}$$

$$\frac{a_4}{a_3} = \frac{30}{20} = \frac{3}{2}$$

$$\frac{a_4}{a_3} = \frac{42}{30} = \frac{7}{5}$$

Since, the common ratio is not constant, it is not an arithmetic sequence.

TRY THIS!

Identify the geometric sequence.

11. $27, 9, 3, 1, \frac{1}{3},...$

12. $-1, 2, -4, 8, -16,...$

Nth TERM OF GEOMETRIC SEQUENCE

The n^{th} term of geometric sequence with first term a_1 and common ratio r is given by: $a_n = a_1 r^{n-1}$.

Example 2:

a. Write a rule for nth term of the geometric sequence $5, 15, 45, 135,...$

b. Find a_5.

a. A/Q,

$a_1 = 5, r = 3$

Now,

$a_n = a_1 r^{n-1}$

$= 5(3)^{n-1}$

Ans: $a_n = 5(3)^{n-1}$

b. $a_5 = 5(3)^{5-1} = 5(3)^4 = 5 \times 81 = 405$

Ans: $a_5 = 405$

TRY THIS!

Write a rule for the nth term of the geometric sequence, and find a_9.

13. $3, 15, 75, 375...$

SUM OF FINITE GEOMETRIC SEQUENCE

The expression formed by adding geometric sequences is called geometric series. Sum of the first n terms of a geometric series is denoted by S_n.

Sum of finite geometric series: $S_n = a_1(\frac{1-r^n}{1-r})$ $\quad r \neq 1$

Example 3:

Find the sum of geometric series $\sum_{i=1}^{10} 4(3)^{i-1}$.

A/Q,

$a_1 = 4(3)^{1-1} = 4(3)^0 = 4 \times 1 = 4$

$r = 3$

Now,

$S_n = a_1(\frac{1-r^n}{1-r})$

$S_{10} = 4(\frac{1-3^{10}}{1-3}) = 4(29524) = 118096$

Ans: $S_{10} = 118096$

TRY THIS!

Find the sum of the geometric series.

14. $\sum_{i=1}^{8} 5^{i-1}$

15. $\sum_{i=1}^{12} 6(-2)^{i-1}$

SUM OF INFINITE GEOMETRIC SEQUENCE

Sum S_n of the first n terms of an infinite series is called partial sum and the infinite geometric series may approach a limiting value.

Sum of infinite geometric series: $S_\infty = \frac{a_1}{1-r}$ $|r| < 1$

NOTE*: If $|r| \geq 1$, then the series has no sum.

Example 4:

Find the sum of infinite geometric series $\sum_{i=1}^{\infty} 3(0.7)^{i-1}$.

A/Q,

$a_1 = 3(0.7)^{1-1} = 3(0.7)^0 = 3 \times 1 = 3$

$r = 0.7$

Now,

$S_\infty = \frac{a_1}{1-r}$

$S_\infty = \frac{3}{1-0.7} = 10$

Ans: $S_\infty = 10$

TRY THIS!

Find the sum of the infinite geometric series.

16. $\sum_{i=1}^{\infty} (-\frac{1}{2})^{i-1}$

17. $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \ldots$

18. $1 + 3 + 9 + 27 + \ldots$

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RECURSIVE RULE FOR SEQUENCE

A **recursive rule** gives the beginning term(s) of a sequence and a recursive equation that tells how a_n is related to one or more preceding terms.

Example 1:

Write the first six terms of the sequence $a_0 = 1,\ a_n = a_{n-1} + 4$**.**

A/Q,

$a_0 = 1$

$a_1 = a_{1-1} + 4 = a_0 + 4 = 1 + 4 = 5$

$a_2 = a_{2-1} + 4 = a_1 + 4 = 5 + 4 = 9$

$a_3 = a_{3-1} + 4 = a_2 + 4 = 9 + 4 = 13$

$a_4 = a_{4-1} + 4 = a_3 + 4 = 13 + 4 = 17$

$a_5 = a_{5-1} + 4 = a_4 + 4 = 17 + 4 = 21$

Ans: $1,\ 5,\ 9,\ 13,\ 17,\ 21.$

TRY THIS!

Write the first seven terms of the sequence.

19. $a_1 = 3,\ a_n = a_{n-1} - 7$
20. $f(0) = 1,\ f(n) = f(n - 1) + n$

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RECURSIVE RULE FOR ARITHMETIC AND GEOMETRIC SEQUENCE

- **Arithmetic Sequence**

  $a_n = a_{n-1} + d$, where d is the common difference.

- **Geometric Sequence**

  $a_n = r \bullet a_{n-1}$, where r is the common ratio.

Example 1:

Write a recursive rule for the sequence $3, 13, 23, 33, 43, ...$

A/Q,

n	1	2	3	4	5
a_n	3	13	23	33	43

+10 +10 +10 +10

The sequence is arithmetic sequence with $a_1 = 3,\ d = 10$

We know,

$a_n = a_{n-1} + d$

$= a_{n-1} + 10$

Ans: The recursive rule for the arithmetic sequence is
$a_1 = 3,\ a_n = a_{n-1} + 10.$

Example 2:

Write a recursive rule for the sequence $16, 40, 100, 250, 625$**....**

A/Q,

n	1	2	3	4	5
a_n	16	40	100	250	625

The sequence is geometric sequence with $a_1 = 16,\ r = \frac{5}{2}$

We know,

$$a_n = r \bullet a_{n-1}$$

$$= \frac{5}{2}a_{n-1}$$

Ans: The recursive rule for the arithmetic sequence is
$a_1 = 16,\ a_n = \frac{5}{2}a_{n-1}$.

TRY THIS!

Write a recursive rule for the sequence.

21. $2, 14, 98, 686, 4802,$...

22. $11, 22, 33, 44, 55,$...

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QUICK REVIEW: SEQUENCE AND SERIES

<table>
<tr><th>Sl. no.</th><th></th></tr>
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A sequence can be defined as an ordered list of numbers sequentially.

● Finite sequence is a function with limited or countable number of terms with domain of first n positive integers {1, 2, 3, …, n} and range $\{a_1, a_2, a_3, ..., a_n\}$.
● Infinite sequence is a function with limitless or uncountable number of terms with domain of all positive integers {1, 2, 3, …, ∞} and range $\{a_1, a_2, a_3, ..., a_\infty\}$.

NOTE*: The domain of a sequence may begin with 0 instead of 1. When this is the case, the domain of a finite sequence is the set {0, 1, 2, 3, . . . , n} and the domain of an infinite sequence becomes the set of nonnegative integers. Unless otherwise indicated, assume the domain of a sequence begins with 1.</td></tr>
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SUM OF ARITHMETIC SEQUENCE: The expression formed by adding arithmetic sequences is called arithmetic series. Sum of the first n terms of an arithmetic series is denoted by S_n.

Sum of finite arithmetic series: $S_n = n(\frac{a_1+a_n}{2})$</td></tr>
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	SUM OF FINITE GEOMETRIC SEQUENCE: The expression formed by adding geometric sequences is called geometric series. Sum of the first n terms of a geometric series is denoted by S_n. Sum of finite geometric series: $S_n = a_1(\frac{1-r^n}{1-r})$ $r \neq 1$ **SUM OF INFINITE GEOMETRIC SEQUENCE**: Sum S_n of the first n terms of an infinite series is called partial sum and the infinite geometric series may approach a limiting value. Sum of infinite geometric series: $S_\infty = \frac{a_1}{1-r}$ $\lvert r \rvert < 1$ NOTE*: If $\lvert r \rvert \geq 1$, then the series has no sum.
4.	**RECURSIVE RULE FOR SEQUENCE**: A **recursive rule** gives the beginning term(s) of a sequence and a recursive equation that tells how a_n is related to one or more preceding terms. **RECURSIVE RULE FOR ARITHMETIC AND GEOMETRIC SEQUENCE** **Arithmetic Sequence**: $a_n = a_{n-1} + d$, where d is the common difference. **Geometric Sequence**: $a_n = r \bullet a_{n-1}$, where r is the common ratio.
5.	**SERIES** A **series** can be defined as a resulting expression when the terms of a sequence are added together. • **Finite series** is a function with sum of limited or countable number of terms expressed as $\sum_{i=1}^{n} a_i$. • **Infinite series** is a function with sum of limitless or uncountable number of terms expressed as $\sum_{i=1}^{\infty} a_i$.

SIGMA NOTATION FOR SERIES: For both finite and infinite series, the index of summation is i and the lower limit of summation is 1. The upper limit of summation is 4 for the finite series and ∞ (infinity) for the infinite series. Summation notation is also called sigma notation because it uses the uppercase Greek letter sigma, written $\sum$.

SUM OF SERIES*:* A series of numbers can be added beginning from the lower limit till the upper limit.

Formulae for Special Series	
Sum of n terms of 1	$\sum_{i=1}^{n} 1 = n$
Sum of first n positive integers	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
Sum of squares of first n positive integers	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$